Disorientation of Suprathermally Rotating Grains and the Grain Alignment Problem

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ABSTRACT

We discuss the dynamics of dust grains subjected to uncompensated torques arising from H_2 formation. In particular, we discuss grain dynamics when a grain spins down and goes through a "crossover". As first pointed out by Spitzer & McGlynn (1979), the grain angular momentum before and after a crossover event are correlated, and the degree of this correlation affects the alignment of dust grains by paramagnetic dissipation. We calculate the correlation including the important effects of thermal fluctuations within the grain material. These fluctuations limit the degree to which the grain angular momentum $\mathbf{J}$ is coupled with the grain principal axis $\mathbf{a}_1$ of maximal inertia. We show that this imperfect coupling of $\mathbf{a}_1$ with $\mathbf{J}$ plays a critical role during crossovers and can substantially increase the efficiency of paramagnetic alignment for grains larger than 10^{-5} cm. As a result, we show that for reasonable choices of parameters, the observed alignment of $a \gtrsim 10^{-5}$ cm grains could be produced by paramagnetic dissipation in suprathermally rotating grains, if radiative torques due to starlight were not present. We also show that the efficiency of mechanical alignment in the limit of long alignment times is not altered by the thermal fluctuations in the grain material.

Subject headings: ISM: Magnetic field, Dust, Extinction – Polarization

1. Introduction

Understanding of the observed alignment of interstellar grains is a challenging problem of nearly a half century's standing (see Roberge 1996). Lacking a proper understanding of the alignment processes, we can only tentatively interpret polarimetric data in terms of the magnetic field. Indeed, polarizing grains can be aligned with long axes either perpendicular or parallel to the magnetic field, depending on what causes the alignment (see Lazarian 1994); they can also be not aligned at all (see Goodman 1996).

One of the essential features of grain dynamics in the diffuse interstellar medium (henceforth ISM) is suprathermal rotation (Purcell 1975, 1979). Originally, three separate causes of suprathermal rotation were suggested: inelastic scattering of impinging atoms when the gas and grain temperature differ, photoelectric emission, and H_2 formation on grain surfaces. The latter was shown to dominate the other two for typical conditions in the diffuse ISM (Purcell 1979). More recently, radiative torques due to starlight have been identified as a major mechanism driving suprathermal rotation (Draine & Weingartner 1996, 1997).

Alignment of grains rotating suprathermally differs considerably from the alignment of thermally rotating grains.[1] The theory of paramagnetic alignment of suprathermal grains was discussed by Purcell (1979) and Spitzer & McGlynn (1979); henceforth SM), and has been elaborated by Lazarian (1995b,c, 1996b). Until recently, mechanical alignment, e.g. alignment caused by a gaseous flow, was thought not to be applicable to suprathermally rotating grains, as rapid rotation makes the grains not susceptible to such a process. However, two new mechanisms of mechanical alignment of suprathermally rotating grains, namely, the "crossover" and "cross-section" mechanisms, have been suggested recently (Lazarian 1995d) and shown to be effective in interstellar regions with gas-grain streaming (Lazarian & Efroimsky 1996; Lazarian et al. 1996).

The crossover event is the most important period in the dynamics of suprathermally rotating grains. The H_2 formation sites on a grain surface have a finite "lifetime" t_L, which may be determined by the "resurfacing" of the grain by accreted atoms (SM, Purcell 1979) or poisoning of active sites by oxygen (Lazarian 1995c). Because of the changes in the resulting torque, the spin-up has a finite duration and this limits the paramagnetic alignment attainable. In the case of mechanical alignment, it is during the crossovers that the grain is susceptible to alignment due to gaseous bombardment.

The pioneering study of SM showed that the direction of angular momentum before and after crossover are correlated. However, SM found that the correlation was insufficient for the paramagnetic mechanism to achieve significant alignment within the model they adopted.

Recent progress in the understanding of certain subtle issues of grain dynamics has led us to reexamine the crossover process. SM assumed that during suprathermal rotation

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[1] See Davis & Greenstein (1951), Jones & Spitzer (1967), Mathis (1986), Roberge et al. (1993), Lazarian (1995a) for paramagnetic alignment of thermally rotating grains; and Gold (1951), Lazarian (1994), Roberge et al. (1995), Lazarian (1996a) for mechanical alignment of thermally rotating grains.

the grain angular momentum is perfectly aligned with the axis of major inertia[2]. This coupling arises from internal dissipation[3] which, as known from theoretical mechanics, causes a spinning solid body to rotate about its axis of major inertia, this being the state of minimum rotational kinetic energy for fixed angular momentum. The assumption of perfect relaxation seems natural, as the time-scale for internal relaxation for suprathermally rotating grains is many orders of magnitude less than the time-scale of the spin-up, but it is not exact: it disregards thermal fluctuations within the grain body.

In fact, it was shown in Lazarian (1994) that due to thermal fluctuations, the coupling above is never perfect (the formal theory of this phenomenon is elaborated in Lazarian & Roberge 1996). The component of angular momentum perpendicular to the axis of the major inertia, although tiny compared to the grain angular momentum during suprathermal rotation, is very important in the course of a crossover. We therefore reconsider the SM theory of crossovers in order to allow for the effects of thermal fluctuations.

In §2 we pose the problem and present the necessary facts concerning incomplete internal relaxation. In §3 we derive the disorientation factor accounting both for thermal fluctuations within the grain material and for the effects of gaseous bombardment. The latter effect is of secondary importance in diffuse clouds but may be important in molecular clouds. The consequences of the incomplete disorientation on paramagnetic and mechanical alignment are discussed in §4 and the conclusions are presented in §5.

2. The problem

A crossover is the event that occurs between two sequential spin-ups when the component of $\mathbf{J}$ parallel to the axis of major inertia passes through zero. This is a critical period for grain dynamics, and during the crossover the grain is susceptible to disorientation, which will limit the effectiveness of paramagnetic alignment. If the grain is situated in a region with substantial gas-grain streaming, the grain is susceptible to mechanical alignment during crossovers. Our task in the present paper is to describe the evolution of grains through crossover events, accounting for the effects of thermal fluctuations within the grain material.

To understand the crossover one needs to recall certain basic features of suprathermal rotation. Here we assume that grains are spun-up by torques arising from H_2 formation, and

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[2]For brevity, we refer to the principal axis of largest moment of inertia as the “axis of major inertia”.

[3]The most important internal dissipation process is Barnett relaxation (Purcell 1979).

consider a "brick" with dimensions $b \times b \times a$ and density ρ_s. The ratio $r \equiv b/2a$ determines the degree of grain oblateness; $r = 1$ for the grain $2:2:1$ discussed in Purcell (1979). It is possible to show (SM; Draine & Lazarian 1996) that the components of the torque perpendicular to the axis of major inertia average out and therefore only the component of the torque parallel to this axis matters. We direct the z-axis along the axis of major inertia. We let

$$n_{\rm H} \equiv n({\rm H}) + 2n({\rm H}_2) \quad , \tag{1}$$

where $n({\rm H})$ and $n({\rm H}_2)$ are the concentrations of atomic and molecular hydrogen, respectively; the H_2 fraction we denote $y \equiv 2n({\rm H}_2)/n_{\rm H}$.

The number of H_2 molecules ejected per second from an individual site is $\gamma a^2 n_{\rm H} v_{\rm H}(1-y)r(r+1)\nu^{-1}$, where γ is the fraction of H atoms (with mean speed $v_{\rm H}$ and mass $m_{\rm H}$) adsorbed by the grain, and ν is the number of active sites over the grain surface. The mean square torque from H_2 formation is (see Appendix A)

$$\langle L_z^2 \rangle \approx \frac{2}{3}\gamma^2 a^6 (1-y)^2 n_{\rm H}^2 m_{\rm H} v_{\rm H}^2 E \nu^{-1} r^4 (r+1) \quad , \tag{2}$$

where $E \approx 0.2$ eV is the kinetic energy of a nascent H_2 molecule. The fluctuating torque L_z spins up grains to an rms angular velocity

$$\langle \omega^2 \rangle^{1/2} = \langle L_z^2 \rangle^{1/2} \frac{t_d}{I_z} \left(\frac{t_L}{t_L + t_d} \right)^{1/2} \quad , \tag{3}$$

(Purcell 1979), where $I_z = \frac{8}{3}\rho_s a^5 r^4$ is the z component of the momentum of inertia, t_d is the rotational damping time (see Appendix A)

$$t_d = \frac{2r}{(r+2)} \frac{a\varrho_s}{n_{\rm H} m_{\rm H} v_{\rm H}} \frac{1}{(1.2 - 0.292y)} \quad , \tag{4}$$

and t_L is the lifetime of an active site.

To obtain both characteristic numerical values and functional dependencies we will use quantities normalized by their standard values (see Table 1). We denote the normalized values by symbols with hats, e.g. $\hat{a} \equiv a/(10^{-5}$ cm), with 10^{-5} cm as the standard value of grain size. We consider 'standard' an H_2 formation efficiency $\gamma = 0.2$ and $v_{\rm H} = 1.5 \cdot 10^5$ cm s^{-1}. For diffuse clouds we assume that all hydrogen there is in atomic form and therefore $y = 0$. Note that sometimes the choice of 'standard' values is somewhat arbitrary, e.g. for the time being we assume the density of active sites[4] $\alpha_{\rm H2}$ to be 10^{11} cm^{-2}, so that $\nu = 80\hat{\alpha}_{\rm H2}\hat{a}^2 r(r+1)$.

[4]The density of active sites depends on the interplay of the processes of photodesorption and poisoning.

Using standard values of the parameters we obtain the following expression for the angular velocity

$$\begin{aligned}\langle\omega^2\rangle^{1/2} &\approx \left(\frac{3E}{\alpha m_{\rm H}}\right)^{1/2}\frac{\gamma}{2a^2}\frac{(1-y)}{(1.2-0.292y)}\frac{1}{r^{3/2}(r+2)}\left(\frac{t_L}{t_d+t_L}\right)^{1/2}\\ &\approx 5.4\cdot 10^9\,\frac{\hat{E}\hat{\gamma}}{\hat{\alpha}^{1/2}\hat{a}^2}\frac{(1-y)}{(1.2-0.292y)}\frac{1}{r^{3/2}(r+2)}\left(\frac{t_L}{t_d+t_L}\right)^{1/2}\ {\rm s}^{-1}.\end{aligned} \tag{5}$$

The lifetime of an active site of H_2 formation is limited by both accretion of a mono-layer of refractory material (SM) and poisoning by atomic oxygen (Lazarian 1995c). The former is usually slower than the latter and could provide long-lived spin up with $t_L \gg t_d$. Further on we use the term "resurfacing" to refer to the fastest mechanism of the two. The component of the mean torque along the axis of major inertia before and after, say, resurfacing may be directed either in the same direction as before the process, or in the opposite direction. In the latter case the grain undergoes a spin-down.

The mean interval between crossovers is (Purcell 1979)

$$\bar{t}_z \approx \pi(t_L t_d)^{1/2} \quad . \tag{6}$$

The active site lifetime t_L is very uncertain. In our numerical examples in this paper we will take $t_L = 10^{12}$ s. For our standard parameters (Table 2) in a diffuse HI cloud this corresponds to $t_L/t_d = 0.25/\hat{a}$, and $t_z \approx 1.6\hat{a}^{1/2}t_d$.

When a grain rotates about an arbitrary axis, the angular velocity precesses in grain body coordinates. The Barnett effect produces a magnetic moment parallel to the angular velocity. Purcell was the first to realize that this should result in internal dissipation with a dissipation time-scale inversely proportional to the angular velocity squared (Purcell 1979). It is possible to show (see §3.2) that this effect suppresses rotation around any axis but the axis of major inertia on a time-scale $t_B \sim 10^7/\aleph$ s, where $\aleph$ is the ratio of grain rotational energy to the equipartition energy $\sim kT$. Both H_2 formation (Purcell 1979) and radiative torques (Draine & Weingartner 1996a) can produce $\aleph > 10^3$ and therefore grains tend to rotate around their axes of major inertia.

It is shown in Lazarian (1995c, 1996b) that the poisoning intensifies when the number of active sites becomes greater than a critical number. The latter number depends on the migration time-scale of hydrogen and the activation barrier for the reaction between physically and chemically adsorbed hydrogen atoms. As the details of the grain chemistry are poorly known the critical number of active sites is highly uncertain. Therefore, for the sake of simplicity, we do not discuss differences in poisoning of active sites whenever their number exceeds the critical number and assume, following SM, a constant surface density α of active sites.

Although the ratio $t_B/\bar{t}_z \approx t_B/t_d$ can be as small as 10^{-5} (see Table 2) the alignment of $\mathbf{J}$ with the axis of major inertia ($\mathbf{a}_1$) is not perfect. The deviations of $\mathbf{a}_1$ from $\mathbf{J}$ arise from thermal fluctuations within the grain material (Lazarian 1994, Lazarian & Roberge 1996). To estimate the value of such deviations recall that rotation about $\mathbf{a}_1$ corresponds to the minimum of the grain kinetic energy for fixed J (internal dissipation does not alter J)[5]. For a symmetric oblate grain with $I_z > I_x = I_y \equiv I_\perp$ (i.e., our "brick" with $b > a$), the grain kinetic energy is

$$E_k(\beta) = \frac{J^2}{2I_z}\left(1 + \sin^2\beta\left(\frac{I_z}{I_\perp} - 1\right)\right) \quad , \tag{7}$$

where β is the angle between $\mathbf{J}$ and $\mathbf{a}_1$. In thermodynamic equilibrium the fluctuations of the kinetic energy should have a Boltzmann distribution:

$$f(\beta)\mathrm{d}\beta = \mathrm{const}\cdot\sin\beta\exp\left(-\frac{E_k(\beta)}{kT_d}\right)\mathrm{d}\beta \quad , \tag{8}$$

where T_d is the dust temperature. It follows from (8) that the fluctuating component of angular momentum perpendicular to the axis of the major inertia $\langle J_\perp^2\rangle \ll J^2$ can be approximated

$$\langle J_{\perp 0}^2\rangle \approx \left(\frac{I_z I_\perp kT_d}{I_z - I_\perp}\right) \quad . \tag{9}$$

We may define the "thermal transverse angular velocity"

$$\omega_{\perp 0} \equiv \frac{\langle J_{\perp 0}^2\rangle^{1/2}}{I_\perp} = \left(\frac{I_z kT_d}{I_\perp(I_z - I_\perp)}\right)^{1/2} = 1.05\times 10^5\frac{\hat{T}_d^{1/2}}{\hat{\rho}^{1/2}\hat{a}^{5/2}}\left(\frac{15}{16r^4 - 1}\right)^{1/2} \quad \mathrm{s}^{-1}. \tag{10}$$

As we will see below, $\omega_{\perp 0}$ is the characteristic value for the minimum value of the grain angular velocity during a crossover.

When the rotation is suprathermal, $\langle J_\perp^2\rangle^{1/2}$ is negligible compared to J and angle β is very close to zero. However, as the component of angular momentum $J_\|$ parallel to the axis of major inertia decreases during crossovers, the angle $\beta = \arctan(J_\perp/J_\|)$ increases.

When the angular velocity decreases sufficiently, internal relaxation becomes less efficient and the value of $\langle J_\perp^2\rangle$ rises as a result of the stochastic character of H_2 formation and impacting gas atoms. At some point the component of $\mathbf{J}$ parallel to $\mathbf{z}$ passes through zero and the grain flips over (SM). Our task is to calculate the correlation of the grain angular momentum before and after the crossover. This is done in the next section.

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[5]If J does change due to H_2 torques one should substitute its value averaged over the time of internal relaxation in Eq.(7).

3. Crossovers

In our treatment below we repeat the reasoning introduced in SM but with allowance for thermal fluctuations. The zeroth approximation, following SM, is the dynamics of a grain subjected to regular torques only. The dynamical effects of the stochastic torques can be evaluated by an approximate theory based on small perturbations of the zeroth-order solution.

3.1. Zeroth approximation

Let $\mathbf{xyz}$ be a coordinate system frozen into the grain, with $\mathbf{z}$ along the grain axis of major inertia $\mathbf{a}_1$. Let $\mathbf{x_0 y_0 z_0}$ be an inertial coordinate system, with $\mathbf{z}_0 \| \mathbf{J}$ (at some initial time). Let β be the angle between the $\mathbf{z}$-axis and $\mathbf{J}$: $J_z = J\cos\beta$. If no external torques act, then $\mathbf{J} =$ constant and the $\mathbf{z}$-axis and the angular velocity $\boldsymbol{\omega}$ will each precess around $\mathbf{J}$ (or $\mathbf{z_0}$) at a frequency $\omega_p = (I_z - I_x)J_z/I_x I_z$.

Now consider the effect of a (weak) torque $\mathbf{L}$ which is fixed in body coordinates $\mathbf{xyz}$. On time scales long compared to ω_p^{-1} the rotation of the grain around $\boldsymbol{\omega}$ and the precession of $\boldsymbol{\omega}$ around $\mathbf{z_0}$ imply that the only torque component which does not average to zero is that due to L_z, the component of $\mathbf{L}$ along the $\mathbf{z}$-axis. After this averaging we obtain

$$\frac{\mathrm{d}\mathbf{J}}{\mathrm{d}t} = L_z \cos\beta \frac{\mathbf{J}}{|J|}. \tag{11}$$

From the Euler equations (see SM) we find the components of $\boldsymbol{\omega}$ in body coordinates

$$\omega_z = \frac{J_z}{I_z} = \frac{L_z t}{I_z}, \tag{12}$$

$$\omega_\perp = J_\perp^2/I_\perp^2 = \mathrm{const}, \tag{13}$$

where $t = 0$ at the moment of crossover. Eq. (11) can be rewritten

$$\frac{\mathrm{d}J_{z_0}}{\mathrm{d}t} = L_z \cos\beta = L_z \frac{J_z}{J_{z_0}} \ . \tag{14}$$

Since $J_z = L_z t$ we obtain

$$J_{z_0}^2 = L_z^2 t^2 + J_\perp^2 \quad . \tag{15}$$

According to (11), the direction of $\mathbf{J}$ does not change – the torque $\mathbf{L}$ acts only to change its magnitude (see Eq. (15)). Thus the zeroth approximation predicts a perfect correlation between the angular momentum directions prior to and after the crossover. The stochastic torques make the story more involved.

3.2. Crossovers & Barnett fluctuations

Our considerations above ignored the fluctuations associated with the Barnett effect. As discussed by Lazarian & Roberge (1996), angle β fluctuates on the time of the Barnett relaxation (Purcell 1979):

$$t_B(\omega) = \frac{A_a}{\omega^2} \quad , \tag{16}$$

where

$$A_a = 7.1 \times 10^{17} \hat{a}^2 \hat{\varrho} \hat{T}_d \hat{K}_0(r) \text{ s}^{-1} \quad . \tag{17}$$

and

$$\hat{K}_0(r) = \frac{3}{125} \frac{(4r^2+1)^3}{r^2(4r^2-1)} \quad . \tag{18}$$

Note that the ratio $r = 1/2$ corresponds to a cubical grain, for which no internal relaxation is expected in agreement with Eq. (17).

The fluctuations in β span the interval $(0, \pi)$ when $J \rightarrow J_{\perp 0}$ (Lazarian & Roberge 1996). SM showed that during crossovers $J \sim J_{\perp}$ and therefore such fluctuations must be accounted for provided that $t_c > t_B(\omega_{\perp})$, where the crossover time is

$$t_c = \frac{2J_{\perp}}{\dot{J}} \quad , \tag{19}$$

where $\dot{J}$ is the time derivative of J.

When $t_B(\omega_{\perp}) \ll t_c$, Barnett fluctuations will cause β to range over the interval $(0, \pi)$, resulting in frequent reversals of the torque in inertial coordinates. Consequently, the actual time spent during the crossover will be increased. Quantitative analysis of this regime is beyond the scope of the present paper; it does appear clear, however, that each crossover will be accompanied by substantial disalignment when $t_B \ll t_c$.

In our present study we confine ourselves to the other limiting case, namely, $t_c/t_B \ll 1$, in which case the Barnett fluctuations during a crossover can be disregarded and the initial distribution of $J_{\perp}$ with the mean value given by Eq. (9) is produced by the Barnett fluctuations during the long time interval between crossovers.

We shall prove below that for typical interstellar conditions the value of $J_{\perp}$ mostly arises from thermal fluctuations within the grain material during intervals of suprathermal rotation and therefore is given by Eq. (9). Thus we can estimate t_c:

$$t_c \approx \frac{2\langle J_{\perp 0}^2 \rangle}{\dot{J}} \approx 2.6 \times 10^9 \left(\frac{\hat{\rho} \hat{T}_d \hat{a} \hat{\alpha}}{\hat{E}} \right)^{1/2} \frac{1}{\hat{\gamma} \hat{n}_H \hat{v}_H (1-y)} \hat{Z}_0(r) \text{ s} \quad , \tag{20}$$

where

$$\hat{Z}_0(r) = \left(\frac{3r(1+4r^2)}{5(4r^2-1)}\right)^{1/2} \quad . \tag{21}$$

The ratio

$$\frac{t_B}{t_c} = \left(\frac{a}{a_c}\right)^{13/2} \quad , \tag{22}$$

where a_c is the critical radius a_c, which for $\omega \approx J_\perp/I_z$ is equal to

$$a_c \approx 1.47 \times 10^{-5} \left(\frac{\hat{T}\hat{\alpha}}{(1-y)^2 \hat{n}_H^2 \hat{v}_H^2 \hat{E} \hat{\rho}_s^3} \hat{K}_1(r)\right)^{1/13} \text{ cm} \quad , \tag{23}$$

where

$$\hat{K}_1(r) \approx \frac{234375 r^5}{(4r^2-1)(4r^2+1)^7} \quad . \tag{24}$$

It follows from our discussion above that we attempt to deal only with the case $a > a_c$ while leaving the more complex regime $a < a_c$, where Barnett fluctuations during the crossover are important, to be dealt with elsewhere. We use the inequality $a > a_c$ rather than $a \gg a_c$ due to the strong dependence of the time ratio t_c/t_B on a: for $a = 10^{-5}$ cm $t_c > 12t_B$, while for $a = 2.0 \times 10^{-5}$ cm $t_B > 7t_c$. Below we analyze the implications of the critical size a_c in the context of the variations of alignment with grain size.

The numerical value $a_c \approx 1.5 \times 10^{-5}$ cm is quite robust: the most uncertain grain parameter is the surface density of active sites α, but even varying α by a factor 10^2 changes a_c by only a factor $10^{2/13} \approx 1.36$.

Although so far we have considered only suprathermal rotation driven by H_2 formation, the existence of a critical size seems to be generic to the problem of disorientation in the course of crossovers.

In our treatment above we disregarded gaseous friction. This is a good approximation in the diffuse medium since $t_c \ll t_d$ (see Table 2). In molecular clouds as $y \rightarrow 1$ the two time scales may become comparable (e.g. if the atomic hydrogen fraction $(1-y)$ drops below 10^{-3}), and gas drag should be included. We also assume $t_L \gg t_c$; in the case of $t_L \ll t_c$ it becomes important to allow for variations in the time-averaged torque L_z during the crossover.

3.3. Random torques

We consider the dynamical evolution given by Eq. (15) as a zeroth-order solution of the problem, and the dynamical effects produced by stochastic torques as perturbations of

this solution.

Each torque event produces an impulsive change of angular momentum $\triangle\mathbf{J} = \triangle J_z\mathbf{z} + \triangle\mathbf{J}_\perp$. The angular deviations of $\mathbf{J}$ in the $(\mathbf{J},\mathbf{z})$-plane are given by

$$\triangle\,\eta_\parallel = \frac{-\triangle\,J_z\sin\beta + \triangle J_{\perp 1}\cos\beta}{J} \tag{25}$$

and in the transverse direction by

$$\triangle\,\eta_\perp = \frac{\triangle J_{\perp 2}}{J} \quad , \tag{26}$$

where $\triangle J_{\perp 1}$ and $\triangle J_{\perp 2}$ are the components of $\triangle\mathbf{J}_\perp$ in the plane parallel and perpendicular to the $(\mathbf{J},\mathbf{z})$-plane, respectively. The grain is subject to the action of various torques. Here we discuss only torques arising from H_2 formation $(\triangle J)_{\mathrm{H}}$ and gaseous bombardment, $(\triangle J)_g$. Let N_1 be the rate of H_2 formation, and N_2 be the rate of gas-grain collisions. To simplify our notation, we denote the mean square change in angular momentum per H_2 formation event

$$(\triangle J_i)^2 = (\triangle J_i)^2_{\mathrm{H}} + \frac{N_2}{N_1}(\triangle J_i)^2_g \quad , \tag{27}$$

where N_2/N_1 is the number of gas-grain collisions per H_2 formation event. The mean quadratic deviation of $\mathbf{J}$ per H_2 formation is

$$\langle\triangle(\eta)^2\rangle = J^{-2}\{\langle(\triangle J_z)^2\rangle\sin^2\beta + \langle(\triangle J_{\perp 1})^2\rangle\cos^2\beta + \langle(\triangle J_{\perp 2})^2\rangle - \langle\triangle J_z\triangle J_{\perp 1}\rangle\sin 2\beta\} \quad , \tag{28}$$

where $\langle\triangle J_z \triangle J_{\perp 1}\rangle = 0$ due to rotation around the axis of major inertia. For a symmetric oblate grain

$$\langle(\triangle J_{\perp 1})^2\rangle = \langle(\triangle J_{\perp 2})^2\rangle = \frac{1}{2}\langle(\triangle J_\perp)^2\rangle \quad , \tag{29}$$

and we obtain

$$\langle(\triangle\eta)^2\rangle = J^{-2}\left\{\langle(\triangle J_z)^2\rangle\sin^2\beta + \langle(\triangle J_\perp)^2\rangle\left(\frac{1+\cos^2\beta}{2}\right)\right\} \quad . \tag{30}$$

The cumulative deflection due to $i+1$ random impulses may be expressed as

$$\cos\eta_{i+1} = \cos\eta_i\cos(\triangle\eta_i) + \sin\eta_i\sin(\triangle\eta_i)\cos\phi_i \quad , \tag{31}$$

where ϕ_i is the angle between the deviation $(\triangle\eta_i)$ and the great circle measured by η_i. Averaging provides us with the result (see SM)

$$\langle\cos\eta_f\rangle = \Pi_i\langle\cos\triangle\eta_i\rangle \quad . \tag{32}$$

As $\triangle\eta$ is small, we can expand $\cos\triangle\eta$ to obtain

$$\Pi_i\langle\cos(\triangle\eta_i)\rangle \approx \Pi_i\left(1-\frac{1}{2}\langle(\triangle\eta_i)^2\rangle\right) \approx \Pi_i \exp\left(-\frac{1}{2}\langle(\triangle\eta_i)^2\rangle\right) \quad , \tag{33}$$

Hence

$$\langle\cos\eta_f\rangle \approx \exp(-F) \quad , \tag{34}$$

where F is the disorientation parameter

$$F \equiv \frac{1}{2}\int_{-\infty}^{+\infty} N_1\langle(\triangle\eta)^2\rangle \mathrm{d}t \tag{35}$$

and $N_1 = L_z/\langle\triangle J_z\rangle$ is the number of H_2 torque events per second. From Eq. (27) and (30) it is seen that both H_2 torques and gaseous bombardment are included in Eq. (35).

To evaluate F, we obtain $\beta(t)$ from the zeroth-order grain dynamics with only regular torques ($J_\perp$ =const, $J_z = L_z t$); from $\tan\beta = J_\perp/J_z$ we obtain

$$\mathrm{d}t = \frac{\mathrm{d}J_z}{L_z} = -\frac{J_\perp \mathrm{d}\beta}{L_z\sin^2\beta} = -\frac{J^2}{J_\perp L_z}\mathrm{d}\beta \quad . \tag{36}$$

Substituting this into Eq. (35) and integrating from $\beta = 0$ to $\beta = \pi$, one obtains

$$F = \frac{\pi}{4}\frac{\langle(\triangle J_z)^2\rangle}{J_\perp|\langle\triangle J_z\rangle|}\left(1+\frac{3}{2}\frac{\langle(\triangle J_\perp)^2\rangle}{\langle(\triangle J_z)^2\rangle}\right) \quad . \tag{37}$$

Using Eq. (27) and (A6-A10), we find

$$F = \frac{179\pi}{224}\frac{\langle(\triangle J_z)^2\rangle}{J_\perp|\langle\triangle J_z\rangle|}\hat{K}_1(r) \quad , \tag{38}$$

where

$$\hat{K}_2(r) = \frac{56}{179}\left[1+\frac{3[16r^2(r+1)+6r+3]}{8r^2(2r+5)}\right] \quad . \tag{39}$$

Eq. (37) was derived assuming $J_\perp$ = const; in fact, it is the value of $J_\perp$ when $\beta \approx \pi/2$ which should be used in Eq. (37). In the pioneering study by SM, it was assumed that $J_\perp$ is initially zero. This assumption is valid only for grain temperatures approaching absolute zero. For nonzero grain temperatures the mean value of this component squared cannot be less than $J_{\perp 0}^2$ given by Eq. (9). To account for this non-zero component, while avoiding solving the corresponding Fokker-Planck equation, in our simplified treatment here we consider the evolution of the difference $\langle J_\perp^2 - J_{\perp 0}^2\rangle$, using the lucid approach suggested in SM.

In deriving Eq. (38) we assumed $J_\perp$ = const. Recognizing now that $J_\perp$ will be time-dependent, we note that disorientation of the grain depends primarily on the value of $J_\perp^{-1}$ near the time of crossover. We therefore seek to establish $\langle J_\perp^2(0)\rangle^{1/2}$, the value at the moment ($t=0$) of crossover. One can write

$$\frac{\mathrm{d}\langle J_\perp^2 - J_{\perp 0}^2\rangle}{\mathrm{d}t} = N_1\langle(\triangle J_\perp)^2\rangle - \frac{2\langle J_\perp^2 - J_{\perp 0}^2\rangle}{t_B} \quad , \tag{40}$$

which generalizes eq. 37 in SM. All the time during a crossover, apart from a short interval when the grain actually flips over, $\omega \gg \omega_\perp$ and therefore it is possible to assume that $\omega_z \approx \omega$ (see SM). According to our initial assumption, regular torques dominate the zero-order dynamics. Thus $\mathrm{d}t = (I_z/L_z)\times \mathrm{d}\omega_z = (I_z/N_1\langle\triangle J_z\rangle)\mathrm{d}\omega_z$ follows from Eq. (36). Substituting

$$\zeta = \frac{\langle\triangle J_z\rangle}{I_z[\langle(\triangle J_\perp)^2_{\mathrm{H}}\rangle + N_2/N_1\langle(\triangle J_\perp)^2_g\rangle]}\left(\frac{2I_z}{A_aN_1|\langle\triangle J_z\rangle|}\right)^{1/3}\langle J_\perp^2 - J_{\perp 0}^2\rangle \tag{41}$$

and

$$u \equiv \left(\frac{2I_z}{A_aN_1\langle\triangle J_z\rangle}\right)^{1/3}\omega_z \tag{42}$$

into Eq. (40) gives (SM)

$$\frac{\mathrm{d}\zeta}{\mathrm{d}u} = 1 - \zeta u^2. \tag{43}$$

Therefore for negative ω_z increasing to zero for $t=0$, one gets

$$\zeta(0) = 3^{1/3}\Gamma\left(\frac{4}{3}\right) \tag{44}$$

and

$$\langle J_\perp^2(0)\rangle = J_{\perp,torque}^2 + J_{\perp 0}^2 \quad , \tag{45}$$

where

$$\begin{aligned} J_{\perp,torque}^2 &= \frac{3^{1/3}\Gamma(4/3)}{2^{1/3}}\frac{\omega_{zc}I_z}{|\langle\triangle J_z\rangle|}\left[\langle(\triangle J_\perp)^2_{\mathrm{H}}\rangle + \frac{N_2}{N_1}\langle(\triangle J_\perp)^2_g\rangle\right] \\ &\approx \left(\frac{3}{2}\right)^{1/3}\Gamma\left(\frac{4}{3}\right)(A_aN_1)^{1/3}|\langle\triangle J_z\rangle|^{-2/3}I_z^{2/3}\langle(\triangle J_\perp)^2_{\mathrm{H}}\rangle(1+\chi) \quad . \end{aligned} \tag{46}$$

and

$$\chi \equiv \frac{N_2}{N_1}\frac{\langle(\triangle J_\perp)^2_g\rangle}{\langle(\triangle J_\perp)^2_{\mathrm{H}}\rangle} \quad , \tag{47}$$

$\Gamma(x)$ is the gamma function, and

$$\omega_{zc} = \left(\frac{A_aN_1|\langle\triangle J_z\rangle|}{I_z}\right)^{1/3} \tag{48}$$

is the value of ω_z such that the crossover time $(I_z\omega_z)/(N_1\langle\triangle J_z\rangle)$ equals the relaxation time. During this time the number of torque events is $|\omega_z|I_z/\langle\triangle J_z\rangle$, and the product of this number over the mean squared increment of angular momentum per torque event, i.e. $[\langle(\triangle J_\perp)^2_{\rm H}\rangle + N_2/N_1\langle(\triangle J_\perp)^2_g\rangle]$, provides the estimate for $\langle J^2_\perp - J^2_{\perp 0}\rangle$ in accordance with Eq. (46).

Comparison of Eq. (46) and eq. (42) in SM shows that the mean value of $J^2_\perp$ arising from stochastic torques is increased by a factor $(1+\chi)$, where

$$\chi \approx \frac{2k(T+T_d)}{\gamma E}\left(\frac{1.2-0.293y}{1-y}\right) \approx 0.43\left(\frac{T+T_d}{100K}\right)\left(\frac{0.2{\rm eV}}{E}\right)\hat{\gamma}^{-1}\left(\frac{1.2-0.293y}{1-y}\right) \quad , \tag{49}$$

and T is the gas temperature. In molecular gas with $1-y \ll 1$, χ can be large, but in HI regions $\chi \leq 0.5$. We also note that χ does not depend on grain geometry.

Supersonic drift causes mechanical alignment that we briefly discuss in section 5.2. Here we limit discussion to the case where gaseous bombardment is isotropic during the crossover event.

For typical interstellar conditions, the $J^2_{\perp 0}$ term in Eq. (46) is much more important than the term due to gaseous bombardment. The importance of the Barnett fluctuations relative to the stochastic torques is measured by

$$R^2 = \frac{J^2_{\perp 0}}{J^2_{\perp,torque}} \approx 412\frac{\hat{T}_d^{2/3}}{\hat{\gamma}_1^{1/3}(1-y)^{1/3}\hat{E}^{2/3}\hat{T}^{1/6}{\hat{n}_{\rm H}}^{1/3}\hat{a}^{5/3}\hat{\alpha}^{1/3}}\frac{1}{1+\chi}\hat{K}_3(r) \quad , \tag{50}$$

where

$$\hat{K}_3(r) = \frac{9^{1/3}\times 41r^2}{(4r^2-1)^{2/3}[16r^2(r+1)+6r+3]} \quad . \tag{51}$$

The fact that the latter function tends to infinity for cubic grains ($r = 1/2$) is the consequence of the simplifications within our model. In fact, for cubic grains the perpendicular component of the grain angular moment will be of the order of the overall angular momentum, as pointed out in §2. It is easy to see, that for moderately oblate grains, however, $\langle J^2_\perp(0)\rangle$ is dominated by the term $J^2_{\perp 0}$ arising from thermal fluctuations. As $\chi \sim (1-y)^{-1}$, for small concentration of atomic hydrogen $J^2_{\perp,torque}$ may become important. One of the problems with considering very small concentrations of molecular hydrogen is that disorientation then happens not only during crossovers but also during spin-ups (see Lazarian 1995d) and this requires the theory of suprathermal alignment to be modified. Moreover, according to Eq. (23) for $y \to 1$ only very large grains obey the theory we discuss here.

On estimating the critical size of a_c in Eq. (23) we assumed that $\langle J_\perp^2\rangle = J_{\perp 0}^2$. In general, this is not true and our estimate of a_c should be multiplied by $(1+R^{-2})^{1/5}$. The latter value, however, is ~ 1 according to Eq. (50) and therefore our estimate of the critical size given by Eq. (23) stays essentially unaltered.

Although in the grain frame of reference $J_{\perp,torque}$ and $J_{\perp 0}$ appear very similar, their difference is obvious in the inertial frame. Indeed, $J_{\perp 0}$ that arises from thermal fluctuations within the grain material does not alter the direction of $\mathbf{J}$ in the latter frame. On the contrary, $J_{\perp,torque}$ that arises from gaseous bombardment and stochastic events of H_2 formation does directly affect the direction of $\mathbf{J}$.

As seen from Eq. (50), we expect to have $J_{\perp 0}^2 \gg J_{\perp,torque}^2$; in this limit, the disorientation parameter F (see Eq. (38)) can be obtained:

$$F \approx 9.0\times 10^{-3}\hat{E}^{1/2}\hat{\alpha}^{1/2}\hat{a}^{-1/2}\hat{T}_d^{-1/2}\hat{\rho}_s{}^{-1/2}(1+\chi)\hat{K}_4(r) \quad , \tag{52}$$

where

$$\hat{K}_4(r) = \sqrt{\frac{5}{3}}\frac{8(2r+5)(4r^2-1)^{1/2}}{179r^{1/2}(4r^2+1)^{1/2}}\left(1+\frac{3(16r^2(r+1)+6r+3)}{8r^2(2r+5)}\right) \quad . \tag{53}$$

The disorientation decreases as $r\to 1/2$, which corresponds to a cubic grain.

For the case of $J_{\perp,torque}^2 \gg J_{\perp 0}^2$ we can also obtain an estimate of F:

$$F \approx 0.26\ \hat{\alpha}^{1/3}\hat{\rho}_s{}^{-1/2}\hat{a}^{-4/3}(\hat{n}_{\rm H}\hat{E}\hat{\gamma}_1\hat{T}_d)^{-1/6}\hat{T}^{1/12}\sqrt{1+\chi}\hat{K}_5(r) \quad . \tag{54}$$

where

$$\hat{K}_5(r) = \frac{8}{179}\frac{\sqrt{205}(2r+5)(4r^2-1)^{1/6}r^{1/2}}{3^{1/6}(1+4r^2)^{1/2}(16r^2(r+1)+6r+3)^{1/2}}\left(1+\frac{3(16r^2(r+1)+6r+3)}{8r^2(2r+5)}\right) \quad . \tag{55}$$

This estimate coincides with that in Lazarian (1995c) in the limit of negligible contribution from the gaseous bombardment[6].

4. Implications for the alignment

[6]The difference in the numerical values obtained here and in Lazarian (1995c) stems from the fact that in the latter paper the function F was defined as the average value for an ensemble of grains with varying $J_\perp$ and ν (see §4.1).

4.1. Paramagnetic alignment

Paramagnetic alignment of suprathermally rotating grains – frequently called Purcell alignment – can be described using the equation (Purcell 1979):

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\frac{\sin\theta\cos\theta}{t_r} \quad , \tag{56}$$

where t_r is the time of relaxation time of a grain with volume V in the ambient field B:

$$t_r = \frac{I_z}{B^2 V K} = 6.7 \times 10^{13} \frac{\hat{\varrho}_s \hat{a}^2 r^2 \hat{T}_d}{\hat{B}^2} \quad \mathrm{s}, \tag{57}$$

where $K \approx 1.2 \times 10^{-13} \hat{T}_d^{-1}$ s (Draine 1996).

The solution of the differential equation above is trivial:

$$\tan\theta = \tan\theta_0 \exp(-t/t_r) \quad . \tag{58}$$

If at $t = 0$ grains are initially randomly oriented, then after time t we obtain the Purcell (1979) expression for

$$Q(t) \equiv 3/2([\cos^2\theta] - 1/3) = \frac{3}{2}\frac{1 - (e^\delta - 1)^{-1/2}\arctan\sqrt{e^\delta - 1}}{1 - e^{-\delta}} - \frac{1}{2} \quad , \tag{59}$$

where $\delta \equiv 2t/t_r$ and square brackets denote averaging over grain initial orientations.

Now suppose that grains are randomly oriented following crossovers and let $P(t)dt$ be the probability that a randomly selected grain will have gone a time $t_b \in [t, t+dt]$ since its last crossover event. To obtain the Rayleigh reduction factor (Greenberg 1968)

$$\sigma = \frac{3}{2}(\langle\cos^2\theta\rangle - 1/3) \quad , \tag{60}$$

one needs to average $Q(t)$:

$$\sigma = \frac{\int_0^\infty Q(t)P(t)dt}{\int_0^\infty P(t)dt} \quad . \tag{61}$$

For our simplified treatment we will assume that for *any particular grain* in the ensemble the crossovers happen periodically with period $t_{\max}$. [7] Then

$$P(t_{\max}) = \begin{cases} t_{\max}^{-1}, & t < t_{\max} \ , \\ 0, & t > t_{\max} \ , \end{cases} \tag{62}$$

[7] The theory of alignment for an arbitrary distribution of time intervals between crossovers is given in Draine & Lazarian (1996).

For this distribution the mean time between crossovers (or zero-crossings) is $\bar{t}_z = t_{\rm max}$. Integrating (61) we get

$$\sigma = 1 + \frac{3}{\delta_{\rm max}}\left[\frac{\arctan\sqrt{e^{\delta_{\rm max}}-1}}{\sqrt{e^{\delta_{\rm max}}-1}} - 1\right] \quad , \tag{63}$$

where $\delta_{\rm max} = 2\bar{t_z}/t_r$. For small $\delta_{\rm max}$ Eq. (63) can be expanded

$$\sigma \approx \frac{\delta_{\rm max}}{10} + \frac{\delta^2_{\rm max}}{210} + \frac{\delta^3_{\rm max}}{840} + \ldots \; . \tag{64}$$

Up to now we assumed complete disorientation in the course of a crossover. It is evident from Table 2 that $t_r \gg t_d$ for typical interstellar conditions. As the mean "time back to crossover" for "short-lived spin-up" (e.g. for $t_L < t_d$) is of the order of t_d, paramagnetic alignment is marginal unless the directions of $\mathbf{J}$ before and after crossovers are strongly correlated (SM).

To account for incomplete disorientation SM adopted the following reasoning: consider crossovers that occur at intervals $t_{\rm max}$; then in a time $t_{\rm max}/F$ the disorientation decreases $\cos\eta$ by $1/e$. Thus, according to SM, the effects of incomplete disorientation during crossovers may be approximated by replacing $\bar{t}_z$ by $\bar{t}_z/\min[\langle F\rangle_J, 1]$. Henceforth we use $\langle ..\rangle_J$ to denote averaging over the distribution of $J_\perp$. We remind our reader that up to now we evaluated F for a grain with $J_\perp = \langle J_\perp^2\rangle^{1/2}$.

We conjecture that replacing $\delta_{\rm max}$ in Eq. (63)

$$\delta_{\rm eff} = \frac{2\bar{t}_z/t_r}{(1-\exp(-\langle F\rangle_J))} \tag{65}$$

to obtain

$$\sigma \approx 1 + \frac{3}{\delta_{\rm eff}}\left[\frac{\arctan\sqrt{e^{\delta_{\rm eff}}-1}}{\sqrt{e^{\delta_{\rm eff}}-1}} - 1\right] \quad , \tag{66}$$

may give a better fit than the SM approximation above, as it allows for residual correlation for $\langle F\rangle_J \gtrsim 1$. It is evident that for $\langle F\rangle_J \gg 1$ and $\langle F\rangle_J \ll 1$ our approximation coincides with that in SM.

To study the effects of incomplete disorientation below we use Monte-Carlo simulations to calculate σ for different ratios of $\bar{t}_z/t_r$ and $\langle F\rangle_J$.

To obtain $\langle F\rangle_J$, we require $\langle 1/J_\perp(0)\rangle_J$. To estimate this we note that $\langle J_\perp^2(0)\rangle = \langle J_x^2(0)\rangle + \langle J_y^2(0)\rangle$ and due to the symmetry inherent to the problem

$$\langle J_x^2(0)\rangle = \langle J_y^2(0)\rangle = 0.5\langle J_\perp^2(0)\rangle \quad . \tag{67}$$

For a Gaussian distribution with $\sigma_1^2 = \langle J_x^2(0)\rangle$, one gets

$$\left\langle \frac{1}{J_\perp(0)} \right\rangle_J = \frac{1}{2\pi\sigma_1^2}\int_0^{+\infty} 2\pi r \frac{1}{r}\exp\left\{-\frac{r^2}{2\sigma_1^2}\right\}\mathrm{d}r = \sqrt{\frac{\pi}{2}}\frac{1}{\sigma_1} = \frac{\sqrt{\pi}}{\langle J_\perp^2(0)\rangle^{1/2}} \quad . \tag{68}$$

Thus from Eq. (38) we get

$$\begin{aligned} \langle F\rangle_J &\approx \frac{\pi^{3/2}}{4} K_1(r) \frac{\langle(\triangle J_z)^2\rangle}{|\langle\triangle J_z\rangle| \langle J_\perp^2(0)\rangle^{1/2}} \\ &= 1.60\times 10^{-2}\hat{E}^{1/2}\hat{\alpha}^{1/2}\hat{a}^{-1/2}\hat{T}_d^{-1/2}\hat{\rho_s}^{-1/2}(1+\chi)\hat{K}_5(r)(1+R^{-2})^{-1/2}. \end{aligned} \tag{69}$$

In those simulations we use Eq. (34) to find $\langle\cos\eta_f\rangle$ for a fixed $J_\perp$ and then perform numerical averaging of $\langle\cos\eta_f\rangle$ over a Gaussian distribution of $J_\perp$. We require a distribution function for the stochastic jumps that correspond to $\langle\langle\cos\eta_f\rangle\rangle_J$. We assume the distribution of η to have the form[8]

$$P_1(\eta)\mathrm{d}\eta = C\sin\eta\exp(-\alpha^2\sin^2(\eta/2))\mathrm{d}\eta \quad , \tag{70}$$

where C is the normalization constant:

$$C = \frac{\alpha^2}{2}\frac{1}{1-\exp(-\alpha^2)} \tag{71}$$

and α is the solution of the transcendental equation:

$$\langle\cos\eta_f\rangle = \frac{1+\exp(-\alpha^2)}{1-\exp(-\alpha^2)} - \frac{2}{\alpha^2} \quad . \tag{72}$$

An individual jump over η during a crossover event happens in a random direction and we obtain the final value of $\theta_{i,f}$ after the i-th crossover from the following formulae:

$$\cos\theta_{i,f} = \cos\theta_{i,b}\cos\eta + \sin\theta_{i,b}\sin\eta\cos x \quad , \tag{73}$$

where x is a random variable uniformly distributed over $[0, 2\pi]$ and $\theta_{i,b}$ is the value of the alignment angle just before the i-th crossover. Between crossovers the dynamics of the alignment angle is determined by Eq. (58). Averaging over $P(t_{\max})$ (see Eq. (62)) is also performed to account for the distributions of the times since the last crossover.

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[8] The assumed functional form (70) has the required behavior of $dP_1/d\eta = 0$ for $\eta = 0, \pi$; $P_1 \sim (\sin\eta)$ for $\alpha \to 0$; $P_1 \sim \eta\exp(-\alpha^2\eta^2/4)$ for $\eta \ll 1$.

The results of those calculations are shown in Fig. 2 where we have plotted σ vs $\delta_{\rm eff}$ (defined by Eq. (65)). For each value of $\delta_{\rm eff}$ different symbols correspond to the alignment measures obtained for different values of $\langle F \rangle_J$. The solid line in the same plot corresponds to Eq. (66). In the limit $\langle F \rangle_J \to \infty$ we have complete disorientation, in which case Eq. (66) is an exact result (for periodic crossovers). However, it is evident that Eq. (66) provides a good approximation to the numerical results for finite $\langle F \rangle_J$, at least for periodic crossovers. More general models where the times between crossovers are obtained through Monte-Carlo simulations are studied in Draine & Lazarian (1997).

For typical values of interstellar parameters (see Table 2) one obtains $F \approx 0.014$ (see Eq. 52). Using our earlier estimate $t_z \approx 1.6 t_d \hat{a}^{1/2}$ (for assumed $t_L/t_d = 0.25/\hat{a}$) we get $\delta_{\rm eff} \approx 13.7$ (indicated by an arrow in Fig. 2), for which Eq. (66) gives $\sigma \approx 0.8$. This high degree of alignment is due to the small estimated value of $\langle F \rangle$. In fact, it was argued in Lazarian (1995c) that t_L is expected to be several times greater than t_d for grains with $a > 10^{-5}$ cm and this, by increasing t_z, would further increase the expected alignment.

Although we do not quantitatively discuss here the alignment of grains with $a < a_c \approx 1.5 \times 10^{-5}$ cm we conjecture that the alignment of such grains may be suppressed by the effective disalignment during each crossover (i.e. $\langle F \rangle \gg 1$), due to the variations in the angle β due to Barnett fluctuations when $t_B \ll t_c$ (see §3.2). The strong dependence of t_B/t_c on a (see Eq.(22)) suggests that this may account for the observed lack of alignment of interstellar grains with $a \lesssim 10^{-5}$ (Kim & Martin 1995).

We see, then, that if the only important torques were those due to H_2 formation, gas-grain collisions, and paramagnetic dissipation, we would expect paramagnetic grains in diffuse clouds to be substantially aligned for $a > a_c \approx 1.5 \times 10^{-5}$ cm, and probably minimally aligned for $a < a_c$, at least qualitatively consistent with observations. It has recently been recognized, however, that starlight plays a major role in the dynamics of $a \gtrsim 0.1$ μm grains: the torques exerted by anisotropic starlight (*i*) drive suprathermal rotation (Draine & Weingartner 1996) and (*ii*) can directly act to align $\mathbf{J}$ with the interstellar magnetic field (Draine & Weingartner 1997).

Since we have neglected starlight torques in the present paper, our conclusion for $a \gtrsim 0.1$ μm grains is only preliminary. A study of crossovers incorporating the effects of both H_2 formation and anisotropic starlight is planned. It appears to us highly likely that when both effects are included, the observed lack of alignment of $a \lesssim 0.1$ μm grains, and substantial alignment for $a \gtrsim 0.1$ μm grains, will be explained.

We recall that suprathermal rotation can also be driven by variations of the accommodation coefficient and photoelectric emissivity (Purcell 1979). In a molecular

($y = 1$) region with no ultraviolet light, Purcell's estimate for the torque due to variations in accommodation coefficient leads to $a_c = 3.0 \times 10^{-5}$ cm as the radius for which $t_B = t_c$. However, for this case we also find $\langle F \rangle > 1$ for $a \approx a_c$, so that Purcell alignment will be insufficient unless $\bar{t}_z \geq t_r$.

4.2. Mechanical alignment

It was previously thought that suprathermally rotating grains are not subject to mechanical alignment when the gas is streaming relative to the grain. However, two mechanisms of mechanical alignment of suprathermally rotating grains, namely, "cross-section" and "crossover" alignment, were proposed by Lazarian (1995d). The first process is caused by the fact that the frequency of crossover events depends on the value of the cross-section exposed to the gaseous flux (see Lazarian & Efroimsky 1996 for more details). The second mechanism arises from the substantial susceptibility of grains to alignment by gas-grain streaming during crossover events.

Both mechanical processes are related to the phenomenon of crossovers. Thus our finding of reduced disorientation during crossovers is a new feature that should be incorporated into the discussion of mechanical alignment. As we mentioned earlier, this reduced randomization is valid only for grains with $a > a_c$, where a_c is given by Eq. (23), and therefore no changes of the earlier results are expected for grains with $a < 10^{-5}$ cm. Such grains can be aligned, for instance, by ambipolar diffusion, which favors small grains.

To start with, consider the cross-section mechanism. In Lazarian (1995d) this mechanism was exemplified using a toy model, namely, a flat disc grain which randomly jumps in the course of a crossover from one position, where the surface of the disc is parallel to the flow, to the other position, where the disc surface is perpendicular to the flow. If the probability per unit time of a crossover is proportional to the rate at which atoms arrive at the grain surface, it is easy to see that the grain will spend more time at orientations where the cross section presented to the streaming gas is minimal. Within the toy model above, this corresponds to the position with the surface of the disc parallel to the flow.

In other words, the cross-section mechanism uses the fact that $\bar{t}_z$ is a function of the angle ϕ between the grain axis of major inertia and the direction of the gaseous flow. Roughly speaking, our study above shows that crossovers with disorientation parameter $\langle F \rangle$ and mean time between crossovers $\bar{t}_z$ are equivalent to crossovers with complete disorientation and the mean time between crossovers $\bar{t}_z/(1 - \exp(-\langle F \rangle))$. If F is dominated by thermal fluctuations its dependence on gaseous bombardment vanishes (see Eq. (52))

as does its dependence on ϕ. Thus the only effect of incomplete disorientation during crossovers (as compared to full disorientation) is to increase of alignment time (the time to attain a steady-state) by a factor $(1 - \exp(-\langle F \rangle))^{-1}$.

"Crossover alignment" depends on the ratio of the randomizing torques arising from H_2 formation and aligning torques caused by gaseous bombardment (see Lazarian 1995d). This ratio neither depends on the number of crossovers nor on the time of alignment. The fact that the thermal fluctuations do not change the direction of $\mathbf{J}$ is essential for understanding why this type of alignment is not suppressed in the presence of the incomplete disorientation during crossovers. It is possible to show, however, that the time of alignment increases by a factor $(1 - \exp(-\langle F \rangle))^{-1}$.

In spite of the fact that the measure of the mechanical alignment does not change, our observation that the time required to reach steady state is increased by a factor $(1 - \exp(-\langle F \rangle))^{-1}$ can be important. This is particularly important whenever grain alignment is caused by a transient phenomenon, e.g., a MHD shock. If $\bar{t}_z/(1 - \exp(-\langle F \rangle))$ is much longer than the time of streaming, the alignment of grains with $a > 1.5 \times 10^{-5}$ cm will be marginal[9].

5. Conclusion

We have shown that thermal fluctuations within the grain material limit the extent to which the axis $\mathbf{a}_1$ of major inertia can be aligned with the angular momentum $\mathbf{J}$ in suprathermally rotating grains. Although the fluctuating angle β between $\mathbf{a}_1$ and $\mathbf{J}$ is tiny when the grain is rotating suprathermally, it becomes larger and of critical importance during periods of crossovers. We have proved that for grains with $a > a_c \approx 1.5 \times 10^{-5}$ cm the non-zero component of $\mathbf{a}_1$ perpendicular to $\mathbf{J}$ arising from thermal fluctuations substantially *diminishes* the degree of the randomization of the angular momentum direction in the course of crossovers. If the only torques acting on a grain are those due to gas-grain collisions, H_2 formation, and paramagnetic dissipation, our estimates show that for large ($a \gtrsim a_c$) grains the grain alignment is close to perfect, while small ($a < a_c$) would have only marginal alignment.

If there is gas-grain streaming, the thermal fluctuations increase the time for mechanical alignment for large suprathermally rotating grains, but do not alter the limiting steady

[9]This by no means preclude small grains from being aligned and we believe that the preferential alignment of small grains could be a signature of such an alignment.

state measure of alignment. If the mechanical alignment is caused by Alfvenic waves, it acts in unison with the paramagnetic mechanism to enhance the alignment of large grains. For small grains mechanical alignment due to transient phenomena (e.g. ambipolar diffusion within MHD shocks) can be the dominant cause of alignment.

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A. Some Results for a Square Prism

Here we consider a square prism, with dimensions $b \times b \times a$, density ρ_s, mass $\rho_s b^2 a$, area $4ba + 2b^2$, and moments of inertia $I_z = (8/3)\rho_s a(b/2)^4$ and $I_x = I_y = (1/3)\rho_s a(b/2)^2[a^2 + 4(b/2)^2]$. We let $r \equiv (b/2a)$ and note that $r = 1$ corresponds to the prism grain discussed in Purcell (1979). We consider a hydrogen-helium gas with density $n_{\rm H} \equiv n({\rm H}) + 2n({\rm H_2})$, temperature T, molecular fraction

$$y \equiv \frac{2n({\rm H_2})}{n_{\rm H}} \quad ; \tag{A1}$$

and $n({\rm He})/n_{\rm H} = 0.1$. The square prism has

$$t_M = \frac{\rho_s a}{n_{\rm H} v_{\rm H} m_{\rm H}(1.2 - 0.293y)} \frac{2r}{(r+1)} \quad , \quad t_d = \frac{(r+1)}{(r+2)} t_M \quad , \tag{A2}$$

where t_M is the time for the grain to collide with its own mass of gas, t_d is the rotational damping time (assuming incident atoms to temporarily stick),[10] and $v_{\rm H} = (8kT/\pi m_{\rm H})^{1/2}$ is the mean speed of H atoms. If a fraction γ of impinging H atoms are converted to H_2, then the H_2 formation rate is

$$N_1 = r(r+1)\gamma(1-y) n_{\rm H} v_{\rm H} a^2 \quad . \tag{A3}$$

We assume the grain to be spinning around the z-axis. The prism is assumed to have ν active sites of H_2 formation distributed randomly over the surface. Following Purcell, we assume that newly-formed H_2 molecules depart from each recombination site at a rate N_1/ν, with fixed kinetic energy E but random directions ($dP/d\theta = 2\sin\theta\cos\theta$, where θ is

[10] Purcell & Spitzer (1971) give t_d/t_M for a square prism but their eq.(9) contains a typographical error: the factor $(5s+1)$ should instead be $(4s+1)$. Our r is equal to $1/(2s)$ as defined in Purcell & Spitzer (1971).

with respect to the local surface normal). The $\nu/(1+r)$ sites on the sides of the prism then produce a steady torque L_z with

$$\langle L_z^2\rangle^{1/2} = r^2(r+1)^{1/2}\gamma(1-y)n_{\rm H}v_{\rm H}a^3\left(\frac{2m_{\rm H}E}{3\nu}\right)^{1/2} \quad , \tag{A4}$$

and a mean angular impulse per recombination event $\langle\Delta J_z\rangle$, with

$$|\langle\Delta J_z\rangle| \approx \frac{\langle L_z^2\rangle^{1/2}}{N_1} = \frac{r}{(r+1)^{1/2}}\left(\frac{2m_{\rm H}a^2E}{3\nu}\right)^{1/2} \quad . \tag{A5}$$

Individual H recombination events, occurring at a rate N_1, contribute random angular momentum impulses with

$$\langle(\Delta J_z)^2\rangle_{\rm H} = \frac{1}{3}\frac{r^2(2r+5)}{r+1}m_{\rm H}a^2E \quad , \tag{A6}$$

$$\langle(\Delta J_\perp)^2\rangle_{\rm H} = \frac{16r^2(r+1)+6r+3}{12(r+1)}m_{\rm H}a^2E \quad . \tag{A7}$$

Gas particles impinge at a rate

$$N_2 = 2r(r+1)n_{\rm H}v_{\rm H}a^2(1.05-y+y/\sqrt{8}) \quad , \tag{A8}$$

If impinging particles temporarily stick and then thermally desorb at temperature T_d, then these collision events produce

$$N_2\langle(\Delta J_z)^2\rangle_g = \frac{2}{3}r^3(2r+5)n_{\rm H}m_{\rm H}v_{\rm H}xa^4k(T+T_d)\left(1.2-y+\frac{y}{\sqrt{2}}\right) \quad , \tag{A9}$$

$$N_2\langle(\Delta J_\perp)^2\rangle_g = \frac{1}{6}r(16(r^2(r+1)+6r+3)n_{\rm H}m_{\rm H}v_{\rm H}a^4k(T+T_d)\left(1.2-y+\frac{y}{\sqrt{2}}\right) \quad . \tag{A10}$$

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notation	meaning
$y \equiv 2n(H_2)/n_H$	H_2 fraction
$\gamma \equiv 0.2\hat{\gamma}$	H recombination efficiency
$v_H \equiv (8kT_{gas}/\pi m_H)^{1/2} = 1.5 \times 10^5 \hat{v_H}$ cm s^{-1}	thermal velocity
$a \equiv 10^{-5}$ cm	grain size
$r \equiv b/2a$	grain axis ratio divided by 2
$E \equiv 0.2\hat{E}$ eV	kinetic energy of nascent H_2
$\alpha \equiv 10^{11}\hat{\alpha}$ cm^{-2}	surface density of recombination sites
$\nu \equiv 80r(r+1)\hat{\alpha}\hat{a}^2$	number of recombination sites
$T_d \equiv 15\hat{T}_d$ K	grain temperature
$T \equiv 85\hat{T}$ K	gas temperature
$n_H \equiv 20\hat{n}_H$ cm^{-3}	density of H nucleon
$\varrho_s \equiv 3\hat{\varrho}_s$ g cm^{-3}	solid density
$B \equiv 5\hat{B} \times 10^{-6}$ G	magnetic field

Table 1: Parameters of grains and ambient medium adopted in this paper.

Crossover	$t_c = 2.9 \times 10^9 (\hat{\rho}\hat{T}_d\hat{\nu}\hat{E}^{-1}\hat{a}^{-1})^{1/2}/(\hat{\gamma}\hat{n}_H\hat{v}_H(1-y))\hat{Z}_0(r)$ s
Barnett effect	$t_B = 7.1 \times 10^7 \hat{a}^7 \hat{\rho}_s^2 (\omega_{\perp 0}/\omega)^2 \hat{Z}_1(r)$ s
Gaseous damping	$t_d = 3.3 \times 10^{12} \hat{a}\hat{\rho}_s (\hat{n_H}\hat{v_H})^{-1}(1.2 - y + y/\sqrt{2})\hat{Z}_2(r)$ s
Paramagnetic relaxation	$t_r = 6.7 \times 10^{13} \hat{\rho}_s \hat{a}^2 \hat{T}_d \hat{B}^{-2} \hat{Z}_3(r)$ s

Table 2: Characteristic times involved. We use $K \approx 1.2 \times 10^{-13}\hat{T}_d^{-1}$ for paramagnetic relaxation (Draine 1996). The functions of grain axis ratio $\hat{Z}_i$ $i = 0, 3$ are as follows: $\hat{Z}_0(r) = (6(1+4r^2))^{1/2}/(5(4r^2-1)(r+1))^{1/2}$, $Z_1(r) = (4r^2+1)^2/(25r^2)$, $Z_2(r) = 2r/(r+2)$ and $Z_3(r) = r^2$.

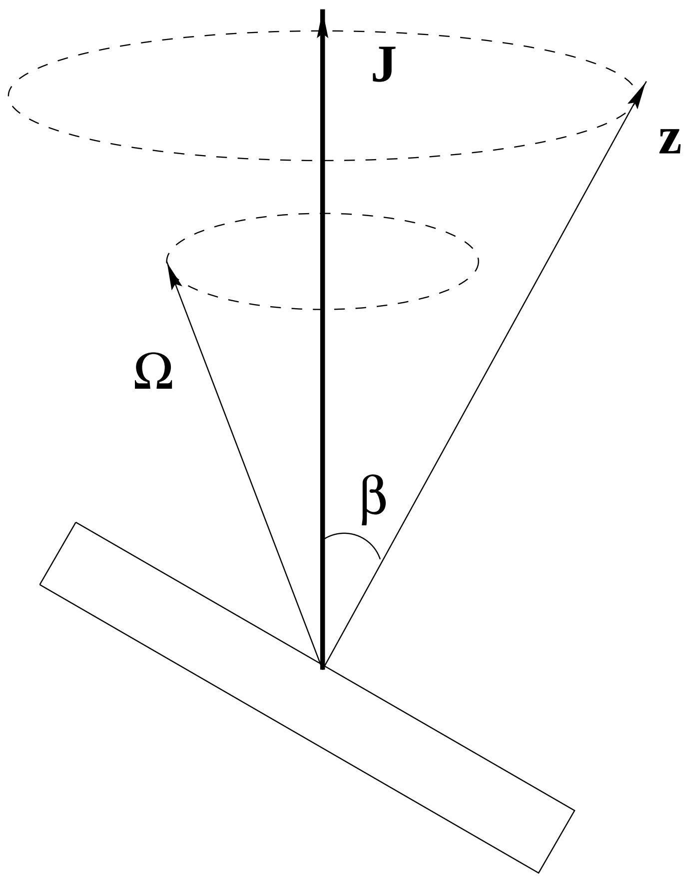

Fig. 1.— Grain's axis of major inertia **z** and **Ω** precess about the direction of angular momentum **J**.

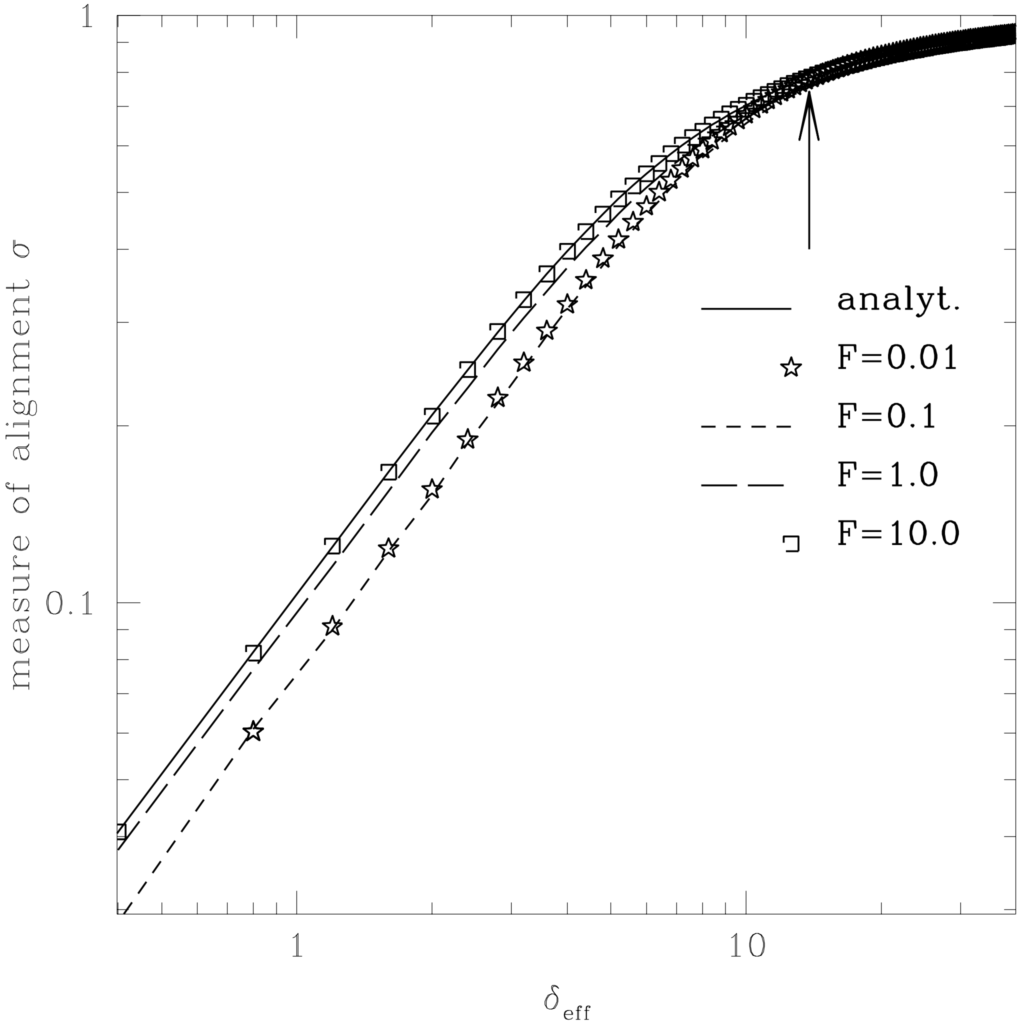

Fig. 2.— Measure of alignment σ [see Eq. (60)] for periodic crossovers as a function of δ_{eff}, defined by Eq. (65). The solid line is the analytic estimate (66).The arrow corresponds to δ_{eff} estimated for typical interstellar conditions (see text).